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# Dealing with Uncertainty

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- The world is not a well-defined place.
- There is uncertainty in the facts we know:
  - What's the temperature today? Imprecise measures
  - Is our Modi good PM? Imprecise definitions
  - Where is the pit? Imprecise knowledge
- People make successful decisions all the time anyhow.

# Sources of Uncertainty

- **Uncertain data**
  - missing data, unreliable, ambiguous, imprecise representation, inconsistent, subjective, derived from defaults, noisy...
- **Uncertain knowledge**
  - Multiple causes lead to multiple effects
  - Incomplete knowledge of causality in the domain
  - Probabilistic/stochastic effects
- **Uncertain knowledge representation**
  - restricted model of the real system
  - limited expressiveness of the representation mechanism
- **inference process**
  - Derived result is formally correct, but wrong in the real world
  - New conclusions are not well-founded (eg, inductive reasoning)
  - Incomplete, default reasoning methods

# Reasoning Under Uncertainty

- So how do we do reasoning under uncertainty and with inexact knowledge?
  - heuristics
    - ways to mimic heuristic knowledge processing methods used by experts
  - empirical associations
    - experiential reasoning
    - based on limited observations
  - probabilities
    - objective (frequency counting)
    - subjective (human experience )

# Decision making with uncertainty

- **Rational** behavior:
  - For each possible action, identify the possible outcomes
  - Compute the **probability** of each outcome
  - Compute the **utility** of each outcome
  - Compute the probability-weighted **(expected) utility** over possible outcomes for each action
  - Select the action with the highest expected utility (principle of **Maximum Expected Utility**)

# Some Relevant Factors

- expressiveness
  - can concepts used by humans be represented adequately?
  - can the confidence of experts in their decisions be expressed?
- comprehensibility
  - representation of uncertainty
  - utilization in reasoning methods
- correctness
  - probabilities
  - relevance ranking
  - long inference chains
- computational complexity
  - feasibility of calculations for practical purposes
- reproducibility
  - will observations deliver the same results when repeated?

# Probability theory

- **Random variables**
  - Domain
- **Atomic event**: complete specification of state
- **Prior probability**: degree of belief without any other evidence
- **Joint probability**: matrix of combined probabilities of a set of variables
- Alarm, Burglary, Earthquake
  - Boolean (like these), discrete, continuous
- $\text{Alarm}=\text{True} \wedge \text{Burglary}=\text{True} \wedge \text{Earthquake}=\text{False}$   
 $\text{alarm} \wedge \text{burglary} \wedge \text{earthquake}$
- $P(\text{Burglary}) = .1$
- $P(\text{Alarm, Burglary}) =$

	alarm	$\neg$ alarm
burglary	.09	.01
$\neg$ burglary	.1	.8

# Belief and Disbelief

- measure of belief
  - degree to which hypothesis H is supported by evidence E
  - $MB(H,E) = 1$  IF  $P(H) = 1$   
 $(P(H|E) - P(H)) / (1 - P(H))$  otherwise
- measure of disbelief
  - degree to which doubt in hypothesis H is supported by evidence E
  - $MB(H,E) = 1$  IF  $P(H) = 0$   
 $(P(H) - P(H|E)) / P(H)$  otherwise



# Certainty Factor

- certainty factor CF
  - ranges between -1 (denial of the hypothesis H) and 1 (confirmation of H)
- $CF = (MB - MD) / (1 - \min(MD, MB))$

# Advantages of Certainty Factors

- Advantages
  - simple implementation
  - reasonable modeling of human experts' belief
    - expression of belief and disbelief
  - successful applications for certain problem classes
  - evidence relatively easy to gather
    - no statistical base required

# Problems of Certainty Factors

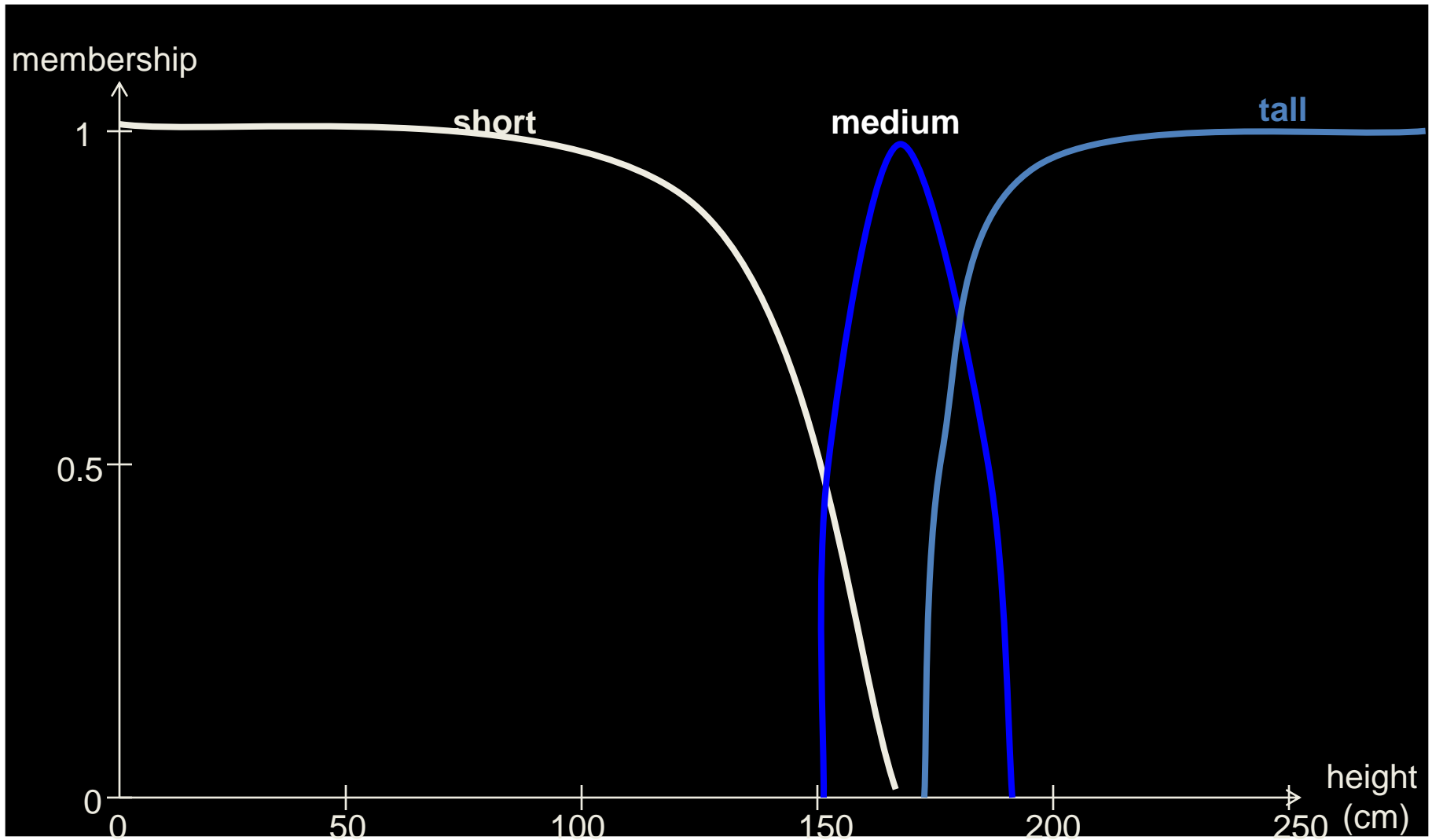
- Problems
  - partially ad hoc approach
    - theoretical foundation through Dempster-Shafer theory was developed later
  - combination of non-independent evidence unsatisfactory
  - new knowledge may require changes in the certainty factors of existing knowledge
  - certainty factors can become the opposite of conditional probabilities for certain cases
  - not suitable for long inference chains

# Fuzzy Logic

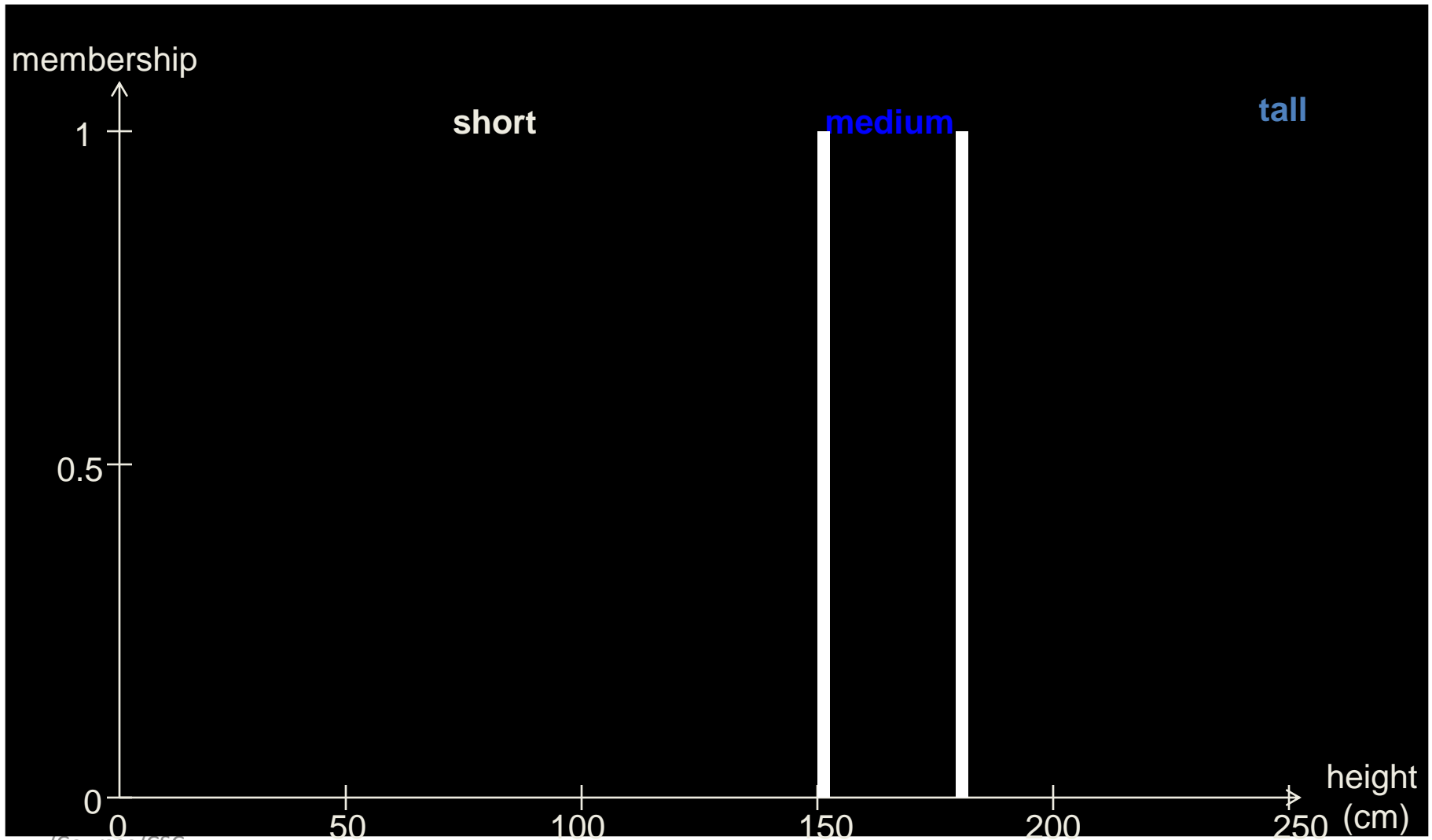
# Fuzzy Logic

- approach to a formal treatment of uncertainty
- relies on quantifying and reasoning through natural (or at least non-mathematical) language
- Rejects the underlying concept of an excluded middle: things have a degree of membership in a concept or set
  - Are you tall?
  - Are you rich?
- As long as we have a way to formally describe degree of membership and a way to combine degrees of memberships, we can reason.

# Fuzzy Set Example



# Fuzzy vs. Crisp Set



# Fuzzy Reasoning

- In order to implement a fuzzy reasoning system you need
  - For each variable, a defined set of values for membership
    - Can be numeric (1 to 10)
    - Can be linguistic
      - really no, no, maybe, yes, really yes
      - tiny, small, medium, large, gigantic
      - good, okay, bad
  - And you need a set of rules for combining them
    - Good and bad = okay.



# Fuzzy Inference Methods

- Lots of ways to combine evidence across rules
  - $\text{Poss}(B | A) = \min(1, (1 - \text{Poss}(A) + \text{Poss}(B)))$ 
    - implication according to Max-Min inference
  - also Max-Product inference and other rules
  - formal foundation through Lukasiewicz logic
    - extension of binary logic to infinite-valued logic
- Can be enumerated or calculated.

# Some Additional Fuzzy Concepts

- Support set: all elements with membership  $> 0$
- Alpha-cut set: all elements with membership greater than alpha
- Height: maximum grade of membership
- Normalized: height = 1

Some typical domains

- Control (subways, camera focus)
- Pattern Recognition (OCR, video stabilization)
- Inference (diagnosis, planning, NLP)

# Advantages and Problems of Fuzzy Logic

- advantages
  - general theory of uncertainty
  - wide applicability, many practical applications
  - natural use of vague and imprecise concepts
    - helpful for commonsense reasoning, explanation
- problems
  - membership functions can be difficult to find
  - multiple ways for combining evidence
  - problems with long inference chains

# Uncertainty: Conclusions

- In AI we must often represent and reason about uncertain information
- This is no different from what people do all the time!
- There are multiple approaches to handling uncertainty.
- Probabilistic methods are most rigorous but often hard to apply; Bayesian reasoning and Dempster-Shafer extend it to handle problems of independence and ignorance of data
- Fuzzy logic provides an alternate approach which better supports ill-defined or non-numeric domains.
- Empirically, it is often the case that the main need is some way of expressing "maybe". Any system which provides for at least a three-valued logic tends to yield the same decisions.