



# Problem & Solution





E  
V  
O  
L  
A I

If  $A = 3 \ \&\& \ 5 < L < O < V < E < 10$



E  
V  
O  
L  
          
A I  
        

If  $A = 3 \ \&\& \ 5 < L < O < V < E < 10$   
Value of I ?



**SRM**

**CSE**

---

**ODD**

---

**Constraints :  $0 \leq \{S, R, M, C, S, E, O, D\} \leq 9$**   
**IF**

**S, R, M = 2, 3, 4**

**What is/are ODD?**

234+721=955

234+621=855



**SRM**  
**CSE**  

---

**GOOD**  

---

**Constraints :  $0 \leq \{S, R, M, C, S, E, O, D\} \leq 9$**   
**IF**

**$(G = 1) \ \&\& \ (S = 7)$**

**How many GOOD(S)?**

$$756+473=1229$$

$$735+274=1009$$

$$734+275=1009$$

$$753+476=1229$$

$$793+872=1665$$

$$792+873=1665$$



LOCK

KEYS

---

DOOR

---

**Constraints :  $0 \leq \{L, O, C, K, E, Y, S, D, R\} \leq 9$**



LOCK  
KEYS  

---

DOOR

Constrains :  $0 \leq \{L, O, C, K, E, Y, S, D, R\} \leq 9$

Satisfactions :  $1638+8027=9665$

Satisfactions :  $2617+7048=9665$

Satisfactions :  $1956+6038=7994$

Satisfactions :  $1956+6042=7998$

Satisfactions :  $2381+1956=4337$

Satisfactions :  $2381+ ? = ?$



# Constraint Satisfaction





# Constraint Satisfaction

- Many AI problems can be viewed as problems of **constraint satisfaction**.



# Constraint Satisfaction

- As compared with a **straightforward search** procedure, viewing a problem as one of **constraint satisfaction** can reduce substantially the amount of search.



# Constraint Satisfaction

- Operates in a **space of constraint sets**.
- Initial state contains the **original constraints** given in the problem.
- A goal state is any state that has been **constrained “enough”**.



# Constraint Satisfaction

## Two-step process:

1. Constraints are discovered and propagated as far as possible.
2. If there is still not a solution, then search begins, adding new constraints.



# Constraint Satisfaction

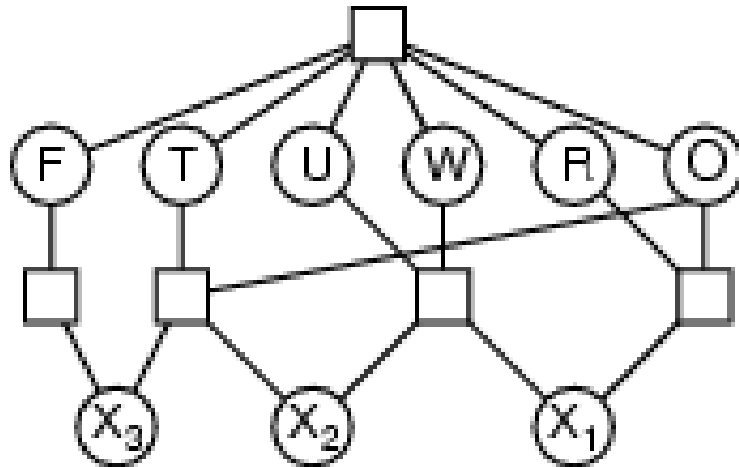
Two kinds of rules:

1. Rules that define valid constraint propagation.
2. Rules that suggest guesses when necessary.

# Constraints

- The simplest type is the **unary constraint**, which constraints the values of just one variable.
- A binary constraint relates two variables.
- Higher-order constraints involve three or more variables. Cryptarithmic puzzles are an example:

$$\begin{array}{r} \text{ T W O} \\ + \text{ T W O} \\ \hline \text{ F O U R} \end{array}$$



# Cryptarithmic puzzles

- **Variables:**  $F, T, U, W, R, O, X_1, X_2, X_3$
- **Domains:**  $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- **Constraints:**
  - Alldiff (F,T,U,W,R,O)
  - $O + O = R + 10 \cdot X_1$
  - $X_1 + W + W = U + 10 \cdot X_2$
  - $X_2 + T + T = O + 10 \cdot X_3$
  - $X_3 = F, T \neq 0, F \neq 0$

$$\begin{array}{r} T \ W \ O \\ + \ T \ W \ O \\ \hline F \ O \ U \ R \end{array}$$



If  $2+2 = 4$ ,  $F=1$ , then how many four

TWO

TWO

---

FOUR

---

$$938+938=1876$$

$$928+928=1856$$

$$867+867=1734$$

$$846+846=1692$$

$$836+836=1672$$

$$765+765=1530$$

$$734+734=1468$$







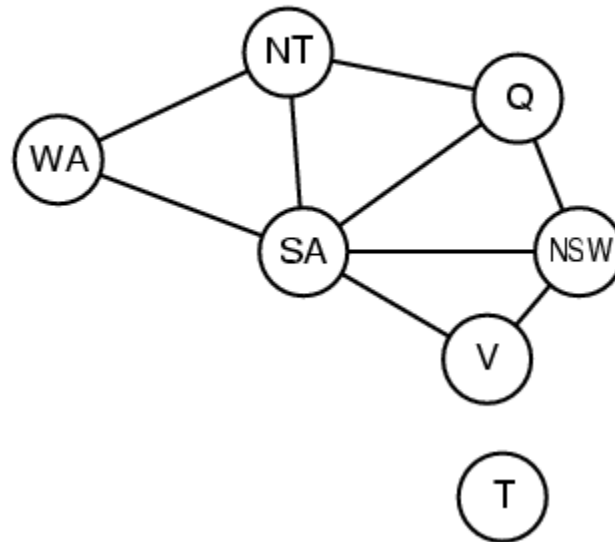
# Map Colo(u)ring



**RED**  
**GREEN**  
**BLUE**



# Map Coloring



# Map Coloring

- **Idea:**
  - Keep track of remaining legal values for unassigned variables
  - Terminate search when any variable has no legal values

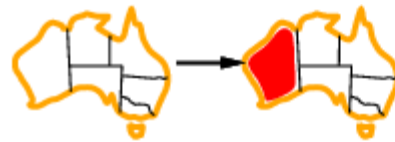


WA      NT      Q      NSW      V      SA      T



# Map Coloring

- **Idea:**
  - Keep track of remaining legal values for unassigned variables
  - Terminate search when any variable has no legal values



WA	NT	Q	NSW	V	SA	T
Red, Green, Blue	Red, Green, Blue	Red, Green, Blue	Red, Green, Blue	Red, Green, Blue	Red, Green, Blue	Red, Green, Blue
Red	Green, Blue	Red, Green, Blue	Red, Green, Blue	Red, Green, Blue	Green, Blue	Red, Green, Blue



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  - Keep track of remaining legal values for unassigned variables
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WA	NT	Q	NSW	V	SA	T
Red, Green, Blue	Red, Green, Blue	Red, Green, Blue	Red, Green, Blue	Red, Green, Blue	Red, Green, Blue	Red, Green, Blue
Red, Red, Red	Green, Blue	Red, Green, Blue	Red, Green, Blue	Red, Green, Blue	Green, Blue	Red, Green, Blue
Red, Red, Red	Blue	Green, Green, Green	Red, Blue	Red, Green, Blue	Blue	Red, Green, Blue



# Map Coloring

- Idea:
  - Keep track of remaining legal values for unassigned variables
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WA	NT	Q	NSW	V	SA	T
Red, Green, Blue	Red, Green, Blue	Red, Green, Blue	Red, Green, Blue	Red, Green, Blue	Red, Green, Blue	Red, Green, Blue
Red, Red, Red	Green, Blue	Red, Green, Blue	Red, Green, Blue	Red, Green, Blue	Green, Blue	Red, Green, Blue
Red, Red, Red	Blue	Green, Green, Green	Red, Blue	Red, Green, Blue	Blue	Red, Green, Blue
Red, Red, Red	Blue	Green, Green, Green	Red	Blue, Blue, Blue		Red, Green, Blue





# Solution?

WA =

NT=

Q=

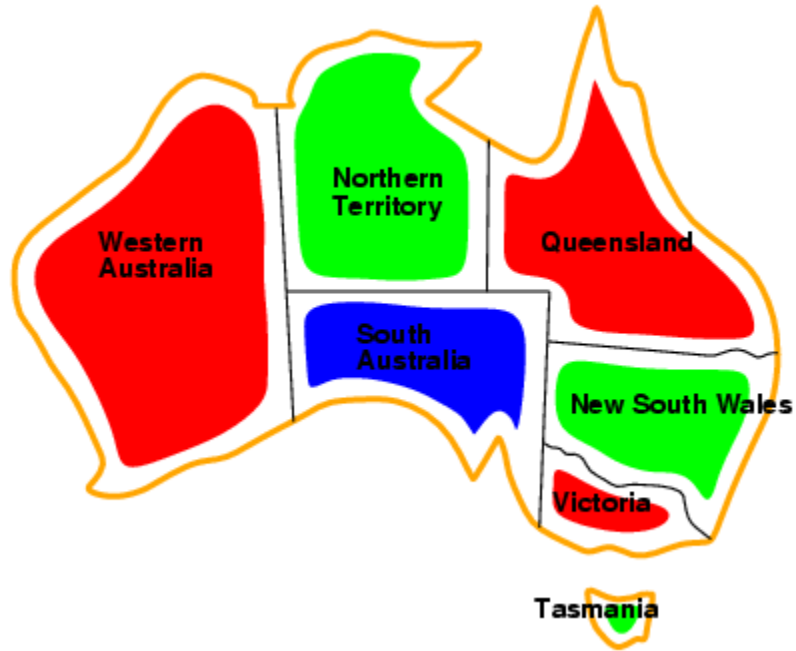
NSW=

V=

SA=

T=







**SEND + MORE = MONEY**

$$\text{SEND} + \text{MORE} = \text{MONEY}$$



1. We have to find values for S,E,N,D,M,O,R,Y (8 digits out of 10). Now, we're adding 2 4-digits numbers. Since  $9999+9999 < 20000$ , M cannot be  $\geq 2$ . And by the "usual rules" for this kind of question, it can't be 0. So  $M=1$ .

$$\text{SEND} + \text{MORE} = \text{MONEY}$$



2. Now, looking at the fourth (left-most) column, we have either  $S+1 \geq 10$  (if there's no carry) or  $1+S+1 \geq 10$  (if there's carry). So  $S=8$  or  $9$ , and  $O=0$  or  $1$ . Since  $1$  is already taken,  $O=0$  (just as well that, or the typography would get confusing).

$$\text{SEND} + \text{MORE} = \text{MONEY}$$



3. In the third column, we can't have  $E+0=N$  (no carry), so  $E+1=N$  and there's carry from the second column. So in the second column either  $N+R=10+E=9+N$ , and  $R=9$ , or there's carry and  $1+N+R=10+E=9+N$ , and  $R=8$ . So  $R=8$  or  $9$ , just like  $S$ .

**SEND + MORE = MONEY**



4. IF S=8 and R=9,  
we're looking at

8END

+ 109E

=====

10NEY

$$\text{SEND} + \text{MORE} = \text{MONEY}$$



5. But this cannot possibly work: we need to get either  $E+0=10+N$  or  $1+E+0=10+N$  in the third column, to get carry in the fourth column. Neither is possible (we've already used up both 9 and 0). So...

$S=9$  and  $R=8$ .

We're looking at

$$\begin{array}{r} 9\text{END} \\ + 108\text{E} \\ \hline 10\text{NEY} \end{array}$$

$$\text{SEND} + \text{MORE} = \text{MONEY}$$



6. We've already used up the digits 0,1,8,9, and  $N=E+1$ , so the only choices for E are 6,5,4,3,2.

7. We know we must have carry from the first column into the second, so  $D+E \geq 10$ . D is at most 7, so we immediately rule out  $E=2$  ( $7+2 < 10$ ). Also  $E=3$  is impossible (because then either  $D=7$  and  $Y=0=O$ , or  $D < 7$  and  $E+D < 10$ , both of which are impossible).



**SEND + MORE = MONEY**



8. If  $E=4$ , then  $D=7$  or  $D=6$  don't work (because then  $Y=1$  or  $Y=0$ , and both are already taken), and  $D<6$  doesn't work because then  $E+D<10$ .

9. If  $E=6$  then  $N=7$ , so  $D\leq 5$ . But  $D=5$  yields  $Y=1$  and  $D=4$  yields  $Y=0$ , both taken, and  $D\leq 3$  gives  $E+D<10$ .

$$\text{SEND} + \text{MORE} = \text{MONEY}$$



10. So  $E=5$  and  $N=6$ .  $D=7$  (the alternative,  $D \leq 4$ , is again too small), so  $Y=2$  and the solution is

$$\begin{array}{r} 9567 \\ + 1085 \\ \hline \hline 10652 \end{array}$$



## Initial state:

- No two letters have the same value.
- The sum of the digits must be as shown.

$$\begin{array}{r} \text{SEND} \\ + \text{MORE} \\ \hline \text{MONEY} \end{array}$$

